Chapter 11

Conclusions

* A summary of main achievements of this thesis.

We have developed a general framework to represent 3D shapes by a hierarchical graph-like medial axis ($\mathcal{MA}$), the *Medial Scaffold* ($\mathcal{MS}$) [125, 87], a hypergraph containing 2D medial sheets, 1D medial curves, and medial vertices. We handle the $\mathcal{MA}$ instabilities as scaffold *transitions* [88], (*i.e.*, topological changes in the graph structure of this scaffold), which are regularized by a set of *transforms* [43], (*i.e.*, graph deformations towards higher symmetries and simplification) [45, 126]. We have also adopted a graph matching approach, the graduated assignment ($\mathcal{GA}$) algorithm to match the $\mathcal{MS}$ hypergraphs. The regularized $\mathcal{MS}$ as a shape representation tool has demonstrated its successfulness in shape recognition and other modeling applications.

* What is special about the medial scaffold?

We point out three significance of the proposed shape representation framework using the $\mathcal{MS}$:

1. *A qualitative $\mathcal{MA}$ structure with a consistent coupled shape.* The major difference of our approach when comparing to other existing 3D shape skeleton extraction methods is that we focus on regularizing medial structures while retaining its approximation to the true $\mathcal{MA}$. In comparison, other methods focus on *pruning* noisy branches and do not simplify their interconnectivity between medial sheets (§ 2.3), and on the other hand, the *curve skeleton* ($\mathcal{CS}$) based methods (§ 2.3.2) over-simplify the medial sheets. Our approach not only regularizes the medial structure but also maintain a consistent shape boundary which is tightly coupled with the $\mathcal{MS}$ (§ 7.3).
2. *An embedded theory of $\mathcal{MA}$ transitions allowing to effectively regularize the $\mathcal{MS}$ and match them with a shape similarity metric.* The structure of the $\mathcal{MS}$ (hypergraph topology, Chapter 4) can be effectively regularized by applying a system of $\mathcal{MS}$ *transforms* (Chapter 5) which essentially moves the $\mathcal{MS}$ toward nearby $\mathcal{MA}$ *transition* points. This approach is in fact embedded under a larger abstract framework to partition the *shape space* by *(i)* grouping similar shapes into a *shape cell* and discretize the *optimal* deformation path to match shapes (Chapter 8).
3. *Computationally practical.* Our approach takes the primitive form of unorganized sample points and re-mesh the surfaces (while additional information such as partial meshes or surface normals are also useful). An initial $\mathcal{MS}$ is obtained during this meshing process and continue to be regularized in our automatic computational pipeline. The regularized $\mathcal{MS}$ hypergraphs are then matched for their similarity using a graph matching scheme (Chapter 9). Our implementation of the $\mathcal{MS}$ are promising in various practical applications (Chapter 10).

* A overview of this thesis *w.r.t.* related line of works of Kimia *et al.*

This thesis continues the approach of Kimia and Leymarie *et al.* by applying the *shock transforms* based on a line of 2D and 3D works: *(i)* the hierarchical organization of the 3D $\mathcal{MA}$ sheets, curves, and isolated points into a *hypergraph* form [87] as well as in a reduced *graph* form [125] toward the notion of the *medial scaffold* ($\mathcal{MS}$), which serves as our representation of the 3D $\mathcal{MA}$; *(ii)* a theoretical study of the *transitions* (sudden topological changes) of the $\mathcal{MA}$ under shape deformations in 2D [89, 200] and in 3D [88]; *(iii)* a practical computational scheme to compute the (full) $\mathcal{MS}$ from unorganized points [125] and produce an initial surface mesh of the shape [45], and further regularize the remaining $\mathcal{MS}$ in an automatic computational system [43, 126]; *(iv)* a shock graph matching framework to match 2D shapes by estimating their optimal deformation guided by the transition of the shock graph ($\mathcal{SG}$) [169], and an approximated graph matching approach using the graduated assignment ($\mathcal{GA}$) to match the $\mathcal{SG}$ [175].

Table 11.1 overviews this thesis with respect to the related works of our group lead by Prof. Kimia *et al.* at LEMS, Brown University, USA.

Table 11.1: A overview of recent $\mathcal{MA}$ and *"shock"* related works Kimia *et al.* in 2D, the *shock graph* ($\mathcal{SG}$), and in 3D, the *medial scaffold* ($\mathcal{MS}$), in terms of theoretical investigations, practical implementations, and related applications.

	2D $\mathcal{MA}$/shocks	3D $\mathcal{MA}$/shocks
Theoretical investigations	$\mathcal{MA}$ local form and transitions [89]. Shape reconstruction from $\mathcal{MA}$ [86].	$\mathcal{MA}$ local form [87]. $\mathcal{MA}$ transitions [88]. $\mathcal{MA}$ consistency conditions [161].
Implementations in practice	$\mathcal{SG}$ computation [195, 202]. $\mathcal{SG}$ transition and regularization [169, 200].	$\mathcal{MS}$ formulation/computation [125]*. $\mathcal{MS}$ regularization [126, 43]*.
Applications	Boundary smoothing [200]. Matching/recognition [169, 175, 148]. Shape generation [203]. Appearance (visual fragments) [196, 149]. Segmentation [170].	Survey of applications [127, 90]. Surface meshing [45]*. Registration [44]*. Feature detection & modeling [43]*. Matching/recognition*.

* works accomplished/addressed in this thesis.

11.1 Conclusive Remarks on Main Topics Covered in this Thesis

* A summary of the $\mathcal{MA}$ and $\mathcal{MS}$, future work of the $\mathcal{SC}$.

The *medial axis* ($\mathcal{MA}$) is the closure of the loci of centers of maximal balls tangent to the object surface at two or more points. A classification of the local form of contact of the ball of tangency leads to ***five*** principal types of $\mathcal{MA}$ points: A_1^2, A_1^3, A_3, A_1^4 and A_1A_3 (Figure 3.2) [87]. The *medial scaffold* ($\mathcal{MS}$) [125] is a hierarchical structure based on this classification of the $\mathcal{MA}$: medial sheets are viewed as "hanging off" a scaffold made from medial curves A_1^3 and A_3 and medial points A_1A_3 and A_1^4). The $\mathcal{MS}$ is a hypergraph of isolated medial points as nodes, medial curves as links, and medial sheets as hyperlinks. The coarse-scale structure of the $\mathcal{MS}$ is represented as a topological hypergraph, and its fine-scale metric is represented as a polygonal mesh.

Future works include a complete shock flow analysis of $\mathcal{MS}$ toward the coarse-scale shock scaffold ($\mathcal{SC}$). As described in § 3.4, this requires to study a topological *surface network* to partition the medial sheets into districts of *monotonic* flows. We believe the proposed dual-scale $\mathcal{MS}$ representation, *i.e.*, the notion of separating topology and fine-scale geometry/dynamics are directly extensible to construct a coarse-scale $\mathcal{SC}$, once a formal understanding of the shock flow on the medial sheets is accomplished.

* Remarks on the $\mathcal{MS}$ transitions and transforms.

Based on a formal study of all *generic* 3D $\mathcal{MA}$ transitions [88], we define the set of $\mathcal{MS}$ transforms covering all cases of the transitions, including the *(i)* generic transitions of simple closed shapes, *(ii)* transitions of non-closed shapes, and *(iii)* non-generic transitions observed in practice. This framework of $\mathcal{MS}$ transforms operates on the *dual-scale* $\mathcal{MS}$ representation (Chapter 4) and effectively regularizes the $\mathcal{MS}$ hypergraph in simplifying its topology and geometry while maintaining a consistent boundary shape. We have also analyzed the high-order degenerate medial nodes of the resulting $\mathcal{MS}$ (Chapter 5).

Future works include to derive more accurate transform cost estimations (§ 8.5) and develop a consistent way to update shape changes for the *interior* contract and merge transforms. We also expect to investigate the $\mathcal{MA}$ transition around the corner shape (§ 5.4.1). In addition, we expect to study the $\mathcal{SC}$ transitions and transforms, which involve additional transitions pertinent to the shock flow change, while the topology of the $\mathcal{MS}$ is keeping intact.

* Remarks on meshing unorganized sample points into surfaces.

We handle unorganized sampled shapes by developing a surface meshing method (Chapter 6) capable of dealing with generic surface topologies: whether they are closed or not, orientable or not, smooth or not, uniformly sampled or not, with non-manifold intersections or not. The input data consists of only 3D positions of (sample) points — no assumptions (on sampling density, normals) are needed to process a raw dataset, although additional knowledge, such as on the sampling density and the local connectivity as a partial mesh, can be used to refine our results. The current implementation is roughly as fast (and with pseudo-linear complexity in the number of samples) as other recent popular methods (see Figure 6.25) and the potential to handle vary large datasets is also very promising. This surface meshing process is also part of the framework of an automatic computation and regularization of the $\mathcal{MS}$ (Chapter 7).

* Remarks on the $\mathcal{MS}$ regularization.

We have developed an approach to stably regularize the $\mathcal{MS}$ by applying the set of transforms to simplify the $\mathcal{MS}$ towards close-by $\mathcal{MA}$ transitions. Our transform-based $\mathcal{MA}$ regularization is drastically different from the traditional approaches which focus on *pruning* medial sheets (see § 2.3). Our approach not only advocates a holistic component-based transformation in pruning spurious medial sheets, but also deal with many additional cases on simplifying the structural interconnectivity between medial sheets, so that the *qualitative* structure of the $\mathcal{MA}$ emerges. Our implementation composes of multiple stages of processes (Chapter 7) to best extract the details of the *"tips"* of the $\mathcal{MA}$ (A_3 ribs) in different cases such as low-sampling and non-solid surfaces. Our system is fully automatic and handles both real-life (scanned) objects, medical models, and degenerate man-made objects.

Future works include the further simplification of the $\mathcal{MS}$ toward an one-dimensional *curve skeleton* ($\mathcal{CS}$). We expect it to provide a solution to relate the $\mathcal{MA}$ to such reduced one-dimensional qualitative structure of the shape.

* Remarks on matching the $\mathcal{MS}$ for 3D shape recognition.

We have developed a 3D shape matching approach to measure 3D shape similarity by matching on their $\mathcal{MS}$ structures (Chapter 9). It is based on the *graduated assignment* algorithm to robustly matching the $\mathcal{MS}$ hypergraphs, while matching the hypergraph nodes/curves/sheets by a matching a set of compatibility functions to reflect both the graph structure and parametric variations of the components. Furthermore, this graph matching scheme can be viewed as an approximated solution embedded under a larger theoretical framework (Chapter 8), that is to view shape deformations as $\mathcal{MA}$ across *transitions* and exploit an optimal *"edit-distance"* algorithm to solve for the "minimal" deformation between two shapes (and thus build a metric in comparing them).

Future works include to exploring the above optimal hypergraph edit-distance matching to match the $\mathcal{MS}$ and the $\mathcal{SC}$, toward solving the general 3D shape recognition problem.